Idempotent matrix: A square matrix A is called idempotent matrix if $A^2 = A$

Example of an idempotent matrix :

$$\text{Let } A = \begin{bmatrix} -1 & 2 & 4 \\ 1 & -2 & -4 \\ -1 & 2 & 4 \end{bmatrix}$$

$$\text{then } A^2 = \begin{bmatrix} -1 & 2 & 4 \\ 1 & -2 & -4 \\ -1 & 2 & 4 \end{bmatrix}^2 = \begin{bmatrix} -1 & 2 & 4 \\ 1 & -2 & -4 \\ -1 & 2 & 4 \end{bmatrix}\begin{bmatrix} -1 & 2 & 4 \\ 1 & -2 & -4 \\ -1 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2-4 & -2-4+8 & -4-8+16 \\ -1-2+4 & 2+4-8 & 4+8-16 \\ 1+2-4 & -2-4+8 & -4-8+16 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 4 \\ 1 & -2 & -4 \\ -1 & 2 & 4 \end{bmatrix}$$

i.e., $A^2 = A$

Exercises:

i. $A = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$	ii. $A = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$	iii. $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$
iv. $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$	v. $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$	vi. $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

$z = x + iy$, where x, y are real numbers and $i = \sqrt{-1}$ is called a complex number.

$\bar{z} = x - iy$ is called the "Conjugate" of the complex number z.

Conjugate of a matrix :-

The matrix obtained by replacing its elements by the corresponding conjugate complex numbers is called conjugate of a matrix.

Transposed conjugate of a matrix:

The transpose of the conjugate of a matrix 'A' is called transposed conjugate of a matrix.

Complex Conjugates of Matrices

original matrix

$$\begin{bmatrix} 2+3i & i & 6-4i \\ 7 & 2-3i & -i \end{bmatrix}$$

complex conjugate

$$\begin{bmatrix} 2-3i & -i & 6+4i \\ 7 & 2+3i & i \end{bmatrix}$$

conjugate transpose

$$\begin{bmatrix} 2-3i & 7 \\ -i & 2+3i \\ 6+4i & i \end{bmatrix}$$

these are not the correct dimensions for multiplication

Hermitian matrix.

A square matrix A is called Hermitian matrix if the transpose of the conjugate of A is equal to A i.e. where $(\bar{A})^T = A$

Show that,

$A = \begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$ is Hermitian.

Solution:

We have to show that, $(\bar{A})^T = A$

So

$\bar{A} = \begin{bmatrix} 4 & 1+3i \\ 1-3i & 7 \end{bmatrix}$

then

$(\bar{A})^T = \begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$

$\Rightarrow (\bar{A})^T = A$

Show that,

$A = \begin{bmatrix} -2 & 2+i & 4 \\ 2-i & 3 & i \\ 4 & -i & 1 \end{bmatrix}$ is Hermitian.

Solution:

We have to show that, $(\bar{A})^T = A$

So

$\bar{A} = \begin{bmatrix} -2 & 2-i & 4 \\ 2+i & 3 & -i \\ 4 & i & 1 \end{bmatrix}$

then

$(\bar{A})^T = \begin{bmatrix} -2 & 2+i & 4 \\ 2-i & 3 & i \\ 4 & -i & 1 \end{bmatrix}$

$\Rightarrow (\bar{A})^T = A$

Exercises:

i. $A = \begin{bmatrix} 1 & 1-i & 2 \\ 1+i & 3 & i \\ 2 & -i & 0 \end{bmatrix}$

ii. . $A = \begin{bmatrix} 2 & 2+i & 4 \\ 2-i & 3 & i \\ 4 & -i & 1 \end{bmatrix}$

iii. $A = \begin{bmatrix} 1 & 2-3i & 3+4i \\ 2+3i & 0 & 4-5i \\ 3-4i & 4+5i & 2 \end{bmatrix}$

iv. Show that, $A = \begin{bmatrix} 1 & 1+i & 2+3i \\ 1-i & 2 & -i \\ 2-3i & i & 0 \end{bmatrix}$ is Hermitian and $\bar{A}$ is Hermitian